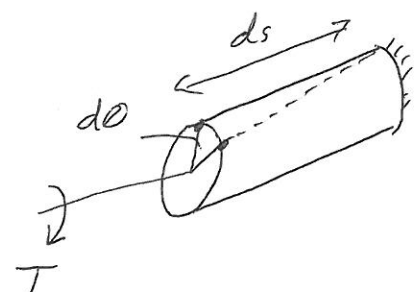




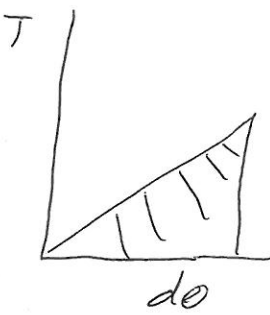
EXAMINATION SOLUTIONS: SESSION 2016/2017

School:	SCHOOL OF ENGINEERING		
Programme:	MECHANICAL & MANUFACTURING	Year:	3
Module/Unit:	ADVANCED STRESS ANALYSIS		
Author:	KELVIN LAKE	Page:	1 of 12

Question No	Question	Marks
1a _i	 SINCE $\frac{T}{J} = \frac{\tau}{R} = \frac{G\phi}{L}$ $\phi = \frac{TL}{GJ}$ ∴ FROM DIAGRAM : $d\phi = \frac{Tds}{GJ}$	(2)

Continuation Sheet

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Question No	Question	Marks
(1a ii)	NOW $U = \frac{1}{2} T d\theta$ ∴ FOR SECTION SHOWN $U = \frac{1}{2} T \cdot \frac{T ds}{GJ}$ $= \frac{T^2 ds}{2GJ}$ ∴ TOTAL STRAIN ENERGY $U = \int_0^L \frac{T^2 ds}{2GJ}$ $= \frac{T^2 L}{2GJ}$ 	(4)
(iii)	RESILIENCE = STRAIN PER UNIT VOLUME IF $U = \frac{P^2 L}{2AE}$ $U = \frac{\sigma^2 A^2 L}{2AE}$ ∴ $U = \frac{\sigma^2 AL}{2E}$ ∴ $U/AL = \frac{\sigma^2}{2E}$ OR $U/V = \frac{\sigma^2}{2E}$	(4)

Continuation Sheet

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Question No	Question	Marks
(6i)	$\sigma = \frac{F}{A} \quad \therefore F = \sigma A = 300 \times 10^6 \times (20 \times 20) \times 10^{-6}$ $= \underline{\underline{120 \text{ kN}}}$ $\text{AREA} = 20 \times 20 \times 10^{-6}$ $= \underline{\underline{4 \times 10^{-4} \text{ m}^2}}$ $\therefore U = \frac{P^2 L}{2AE} = \frac{120,000^2 \times 4}{2 \times 4 \times 10^{-4} \times 200 \times 10^9} = \underline{\underline{360 \text{ J}}}$	(3)
(ii)	$\text{P.E.} = mgh$ $= mg(h + \delta L)$ $E = \frac{\sigma}{\epsilon} = \sigma \div \frac{\delta L}{L} = \frac{\sigma L}{\delta L} \quad \therefore \delta L = \frac{\sigma L}{E}$ P.E. IS CONVERTED INTO STRAIN ENERGY $\therefore mg(h + \delta L) = \frac{P^2 L}{2AE}$ $mg\left(h + \frac{\sigma L}{E}\right) = \frac{\sigma^2 A^2 L}{2AE}$ $\therefore mg\left(h + \frac{\sigma L}{E}\right) = \frac{\sigma^2 AL}{2E}$ $mg\left(h + \frac{\sigma L}{E}\right) = \frac{\sigma^2 V}{2E}$ $\therefore \frac{\sigma^2 V}{2Emg} = h + \frac{\sigma L}{E}$ $\therefore h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$	(4)

Continuation Sheet

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Question No	Question	Marks
(biii)	$h = \frac{\sigma^2 L}{2E\gamma} - \frac{\sigma L}{E}$ $= \frac{(75 \times 10^6)^2 \times (4 \times 10^{-4} \times 4)}{2 \times 200 \times 10^9 \times 200 \times 9.81} - \frac{75 \times 10^6 \times 4}{200 \times 10^9}$ $= \frac{9 \times 10^{12}}{7.85 \times 10^{14}} - \frac{300 \times 10^6}{200 \times 10^9}$ $= 11.47 \times 10^{-3} - 1.5 \times 10^{-3}$ $= \underline{\underline{9.97 \text{ mm}}}$	

Continuation Sheet

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Question No	Question	Marks
(2a)	FIND I_{xx} & I_{zz} $I_{xx} = 2 \times I_{xx \text{ CHANNEL}} + 2 \times I_{xx \text{ PLATE}}$ $= 2 \times 26.2 \times 10^6 + 2 \times \left(\frac{bd^3}{12} + Ah^2 \right)$ $= 2 \times 26.2 \times 10^6 + 2 \left(\frac{300 \times 10^3}{12} + 300 \times 10 \times 105^2 \right)$ $= 52.4 \times 10^6 + 66.2 \times 10^6$ $= \underline{\underline{118.6 \times 10^6 \text{ mm}^4}}$ $I_{zz} = 2 I_{zz \text{ CHANNEL}} + 2 \times I_{zz \text{ PLATE}}$ $= 2(I_{yy} + Ah^2) + 2 \times I_{zz \text{ PLATE}}$ $= 2 \times (1.57 \times 10^6 + 4400 \times 76^2) + 2 \times \frac{10 \times 300^3}{12}$ $= 53.97 \times 10^6 + 45 \times 10^6$ $= \underline{\underline{98.97 \times 10^6 \text{ mm}^4}}$ $\therefore I_{xx} > I_{zz}$ IT WILL BUCKLE ABOUT ZZ	(5)

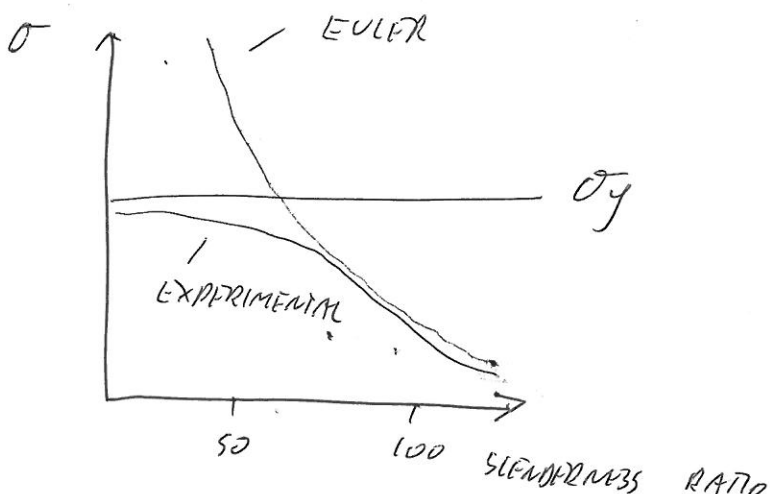
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Question No	Question	Marks
(2b)	$K = \sqrt{\frac{I}{A}}$ $A = 2(4400 + 300 \times 10)$ $= 14800 \text{ mm}^2$ $\therefore K = \sqrt{\frac{98.97 \times 10^6}{14800}} = \underline{\underline{81.8 \text{ mm}}}$	(2)
(2c)	SLENDERNESS RATIO $= \frac{L}{K}$ $= \frac{8}{81.8 \times 10^{-3}}$ $= \underline{\underline{97.9}}$	
(2d)	$P_c = \frac{\pi^2 EI}{L^2}$ $= \frac{\pi^2 \times 200 \times 10^9 \times 98.97 \times 10^{-6}}{8^2}$ $= \underline{\underline{3.05 \times 10^6 \text{ N}}}$	

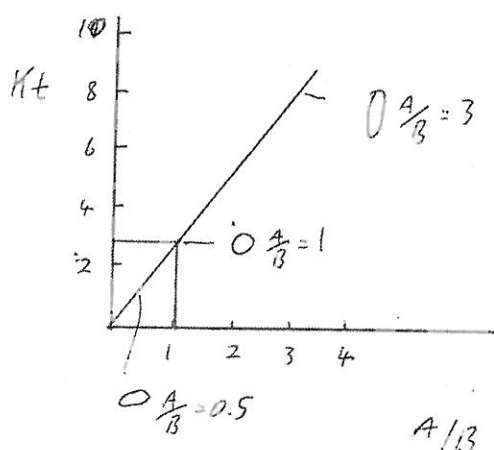
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Question No	Question	Marks
(2e)	$P_n = \frac{\sigma_y A}{1 + \lambda \left(\frac{L}{r}\right)^2}$ $= \frac{320 \times 10^6 \times 0.0148}{1 + \frac{1}{7500} \times 97.9^2}$ $= \underline{\underline{2.08 \times 10^6 \text{ N}}}$	(3)
(f)	FOR THIS STRUT, WHICH IS APPROACHING THE VALIDITY LIMIT FOR EULERS THEORY, THERE IS AN APPRECIABLE DIFFERENCE BETWEEN THE RESULTS GIVEN BY THE TWO THEORIES 	

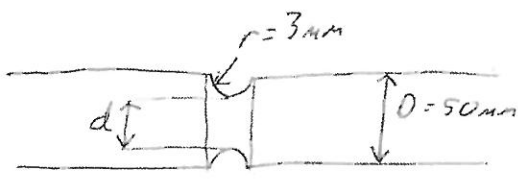
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Question No	Question	Marks
(3) (a)	ITEMS SUCH AS - GROOVES - FILLETS - THREADS - HOLES - KEYWAYS - CONCENTRATED LOADS - MATERIAL PLAYS - ETC SHOULD BE DISCUSSED	(5)
(b)	 IF $\frac{A}{B} = 1$ THEN $K_t = 3$ ie $K_t = 1 + 2\left(\frac{A}{B}\right)$	(6)

Continuation Sheet

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Question No	Question	Marks
C i	 $D = 50 \text{ mm}$ $d = 50 - 6 = 44 \text{ mm}$ $\frac{D}{d} = \frac{50}{44} = \underline{\underline{1.14}}$ $\frac{r}{d} = \frac{3}{44} = \underline{\underline{0.07}}$ $\sigma_{nom} = \frac{4P}{\pi d^2} = \frac{4 \times 50 \times 10^3}{\pi (44 \times 10^{-3})^2} = \underline{\underline{32.9 \text{ MPa}}}$ FROM DIAGRAM $K_t = 2.51$ $\sigma_{max} = 2.51 \times 32.9$ $= \underline{\underline{82.6 \text{ MPa}}}$	

3

Continuation Sheet

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Question No	Question	Marks
(C) (ii)	BENDING $\sigma_{nom} = \frac{32M}{\pi d^3} = \frac{32 \times 150}{\pi \times (44 \times 10^{-3})^3}$ $= \underline{\underline{17.9 \text{ MPa}}}$ FROM DIAGRAM $K_t = 2.24$ $\therefore \sigma_{max} = 2.24 \times 17.9 = \underline{\underline{39.9 \text{ MPa}}}$	(3)
(iii)	TORSION $\tau_{nom} = \frac{16T}{\pi d^3} = \frac{16 \times 320}{\pi \times (44 \times 10^{-3})^3} = \underline{\underline{19.1 \text{ MPa}}}$ FROM DIAGRAM $K_t = 1.65$ $\therefore \sigma_{max} = 1.65 \times 19.1$ $= \underline{\underline{31.5 \text{ MPa}}}$	(3)



(a) A FATIGUE FAILURE IS A FAILURE SEEN UNDER FLUCTUATING STRESS WHICH IS BELOW THE TENSILE STRENGTH OR EVEN THE ELASTIC LIMIT OF THE MATERIAL.

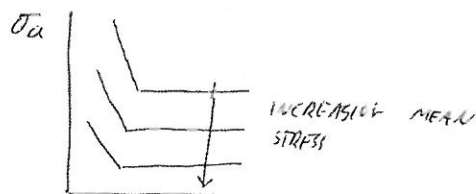
[2 MARKS]

(b) THE SURFACE OF A FATIGUE-FAILED COMPONENT TYPICALLY HAS 3 AREAS:

- (1) THE POINT OF INITIATION
- (2) "ARREST LINES" WHEN THE CRACK PROPAGATES
- (3) THE CRYSTALLINE AREA OF RUPTURE

[3 MARKS]

(c)

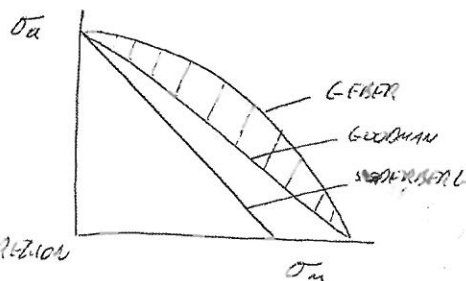


AS THE MEAN STRESS OF A SPECIMEN IS INCREASED, THEN THE FATIGUE LIFE IS REDUCED

(d) THREE THEORIES ARE

[3 MARKS]

- GOODMAN
- GERBER
- SODERBERG



FAILURES ARE TYPICALLY SEEN IN THE SHADED REGION BETWEEN GERBER & GOODMAN.

AS SUCH PEOPLE OFTEN USE SODERBERG AS IT IS THE MOST CONSERVATIVE.

[8 MARKS]



<u>CYCLES SEEN</u>	<u>CYCLES TO FAILURE</u>	
3000 @ 510 MPa	10,050	at $\sigma_m = 0$
12,000 @ 450 MPa	50,500	" "
80,000 @ 380 MPa	275,000	" "
x @ 480 MPa	20,800	" "

(2)

FROM MINERS LAW

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \frac{n_4}{N_4} = 1$$

$$\frac{3000}{10,050} + \frac{12,000}{50,500} + \frac{80,000}{275,000} + \frac{x}{20,800} = 1$$

$$\therefore \frac{x}{20,800} = 1 - \left(\frac{3000}{10,050} + \frac{12,000}{50,500} + \frac{80,000}{275,000} \right)$$

$$= 1 - 0.8270$$

$$x = \underline{\underline{3598}} \text{ CYCLES}$$

[4 MARKS]